Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 27, 2003*
Accepted: *March 5, 2003*

Anti-de Sitter branes with Neveu-Schwarz and Ramond-Ramond backgrounds

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Abstract: We review some facts about $AdS_2 \times S^2$ branes in $AdS_3 \times S^3$ with a Neveu-Schwarz background, and consider the case of Ramond-Ramond backgrounds. We compute the spectrum of quadratic fluctuations in the low-energy approximation and discuss the open-string geometry.

Keywords: D-branes, String Duality.

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Contents

1. Introduction

It was argued in [1] that a "compactification" à la Randall-Sundrum can be implemented in string theory by D5-branes in the near-horizon geometry of D3-branes. One thus obtains generically an $AdS_4 \times S^2$ brane in $AdS_5 \times S^5$. As $AdS_5 \times S^5$ has a nonzero Ramond-Ramond (RR) background field, one is led to consider, more generally, $AdS \times S$ geometries with nonzero RR fluxes.

A simple case is the $AdS_2 \times S^2$ brane in $AdS_3 \times S^3$, because the setup with RR backgrounds is S-dual to the well-known setup with Neveu-Schwarz backgrounds, so we will consider that case here. After computing the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ content of the spectrum, we study the open string geometry of the brane. One reason to study it is the fact, first noticed in [2], and conjectured to be general at least in the probe brane limit, that the anti-de-Sitter and sphere radii are the same, which, in the case of the Karch-Randall setup [1], would make difficult the construction of a realistic model from it. So, we compute the effective radii of the D-brane. We find that:

- The $SL(2,\mathbb{R}) \times \mathrm{SU}(2)$ content of the spectrum is the same in the NS and RR cases, which is expected from S-duality.
- As in the NS case, the two radii are equal.
- Contrary to the NS case, where the common radius is equal to the radius of the bulk $AdS_3 \times S^3$, the common radius in the RR case can take any value.

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The fact that the radii are equal can be shown to be a consequence of supersymmetry (cf [2]). More precisely, this seems to be linked with extended supersymmetry, where the R-symmetry group is the isometry group of a sphere. So one can hope to obtain realistic brane worlds from Randall-Sundrum only from $\mathcal{N} \leq 1$ supersymmetry.

This paper is organized as follows. In section 2, we recall some facts about the Neveu-Schwarz case. The novel results of this article are in section 3. For the reader's convenience, we include the proof of the $\mathrm{SL}(2,\mathbb{R})$ symmetry of the low-energy action of the D3-brane in the appendix, with the transformations of the fields.

2. A reminder of $AdS_2 \times S^2$ D-branes in $AdS_3 \times S^3$ with pure Neveu-Schwarz background

2.1 Notations

We parametrize $AdS_3 \times S^3$ with coordinates such that the metric and the NS two-form read:

$$\begin{aligned} ds^2 &= L^2 \left[d\psi^2 + \cosh^2\psi \left(d\omega^2 - \cosh^2\omega\, d\tau^2\right) + d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] \\ B &= L^2 \left[\left(\psi + \frac{\sinh 2\psi}{2}\right)\cosh\omega\, d\omega \wedge d\tau + \left(\chi - \frac{\sin 2\chi}{2}\right)\sin\theta\, d\theta \wedge d\phi\right] \end{aligned} \tag{2.1}$$

and the dilaton is a constant $\Phi = \Phi_{\mathrm{NS}}$.

Symmetric D-branes in this geometry are hypersurfaces with $\psi = \psi_0$ and $\chi = \chi_0$, so their geometry is $AdS_2 \times S^2$ with induced radii $L\cosh\psi_0$ and $L\sin\chi_0$. They can be stabilized by an appropriate electromagnetic field:

$$F_0 = -\frac{L^2}{2\pi\alpha'}\left(\psi_0 \cosh\omega\, d\omega \wedge d\tau + \chi_0 \sin\theta\, d\theta \wedge d\phi\right). \tag{2.2}$$

Their low-energy effective action is given by the Dirac-Born-Infeld plus Wess-Zumino (DBI-WZ) action, which in the most general case reads

$$S = T_{D3}\left[-\int d^4x e^{-\Phi}\sqrt{-\det(\hat{g} + \mathcal{F})} + \int \left(\hat{C}_{(4)} + \hat{C}_{(2)} \wedge \mathcal{F} + \frac{1}{2}C_{(0)}\mathcal{F} \wedge \mathcal{F}\right)\right], \tag{2.3}$$

where $\mathcal{F} = 2\pi\alpha' F + \hat{B}$. (As $C_{(4)}$ will be zero in this whole article, it will not be mentioned again.)

2.2 Quantized charges

There are two quantized charges:

- $p = -\frac{1}{2\pi}\int_{S^2} d\theta\, d\phi\, F_{\theta\phi}$ is an integer because of Dirac quantization. It can be interpreted as the D-string charge of the D3-brane.

- The F-string charge should, of course, also be an integer; since F-strings couple with the NS two-form, this charge reads

$$q = \frac{1}{T_F}\int_{S^2} d\theta\, d\phi\, \frac{\delta S}{\delta B_{\omega\tau}} \equiv \frac{1}{2\pi}\int_{S^2} d\theta\, d\phi\, \tilde{F}_{\theta\phi}\,; \tag{2.4}$$

  the explicit form of $\tilde{F}$ is given in the appendix.

Notice that these facts are for any D3-brane with topology $M \times S^2$.

In our case, one has $p = \frac{L^2}{\pi\alpha'}\chi_0$ and $q = -\frac{L^2}{\pi\alpha'}e^{-\Phi}\sin\chi_0 \sinh\psi_0$.[1] The fact that these quantities are quantized means two things:

- The set of possible positions for the branes is discrete; in particular, the χ_0 parameter can have only a finite number of values.
- The fluctuations of the branes must be compatible with these quantization conditions; this implies the stability of the branes (see [3]) and some restrictions on the spectrum.

It also implies a restriction of the $SL(2,\mathbb{R})$ symmetry group of the DBI-WZ action to the S-duality group $SL(2,\mathbb{Z})$, which is precisely the subgroup of $\mathrm{SL}(2,\mathbb{R})$ which preserves the quantization. This $\mathrm{SL}(2,\mathbb{Z})$ is conjectured to be an exact symmetry of the full string theory.

2.3 Quadratic fluctuations

The spectrum of quadratic fluctuations was computed in [4]. They are obtained by taking

$$\psi = \psi_0 + \delta\psi\,, \quad \chi = \chi_0 + \delta\chi\,, \qquad \text{and} \qquad F = F_0 + \frac{L^2}{2\pi\alpha'}f \qquad \text{with } df = 0\,, \tag{2.5}$$

and expanding the action up to quadratic terms in $\delta\psi$, $\delta\chi$ and f. The expansion begins with two linear terms that are proportional to $\int d\theta\, d\phi\, f_{\theta\phi}$, which is zero because of the quantization condition, and $\int d\omega\, d\tau\, f_{\omega\tau}$, which is zero if the electromagnetic potential behaves well at infinity because AdS_2 is topologically trivial.

The quadratic terms read:

$$\begin{aligned}\delta^{(2)}S =\; & T_{D3}L^4 e^{-\Phi}\cosh\psi_0 \sin\chi_0 \int d\omega\, d\tau\, d\theta\, d\phi \sqrt{-\det\gamma}\,\times \\ & \times\Big[-\frac{1}{2}\left(\partial_m\delta\psi\partial^m\delta\psi + \partial_m\delta\chi\partial^m\delta\chi\right) - \frac{1}{4}f_{mn}f^{mn} + (\delta\psi)^2 - (\delta\chi)^2 - \\ & - 2\delta\psi\frac{f_{\omega\tau}}{\cosh\omega} + 2\delta\chi\frac{f_{\theta\phi}}{\sin\theta}\Big] - T_{D3}L^4 e^{-\Phi}\sinh\psi_0\cos\chi_0\int f\wedge f\end{aligned} \tag{2.6}$$

where γ is the $AdS_2 \times S^2$ metric with radii 1, and indices are raised with it.

It is then easy to find the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ content of the spectrum, which is explicitly given in [4], where it is also shown that it agrees with the results of conformal field theory (except for the brane-dependent cut-off on the allowed angular momenta).

3. $AdS_2 \times S^2$ D-branes in $AdS_3 \times S^3$ with RR background

3.1 The background

The RR background is obtained from the NS background by the following S-duality transformation: $C_{(2)} \to B$; $B \to -C_{(2)}$; $F \to \tilde{F}$ and $\tau \to -1/\tau$, where $\tau = C_{(0)} + ie^{-\Phi}$ (this corresponds to $(r,s,t,u) = (0,-1,1,0)$ in equation (A.4)). It is shown in the appendix that $\tilde{F}$ is then transformed into $-F$. Thus, the quantum numbers (p,q) become $(-q,p)$, which means that D-string and F-string charge are exchanged.

[1] Notice that the quantization condition given in [2] is true only in the small χ_0 limit.

The fields obtained this way are

$$\begin{aligned}
\Phi &= -\Phi_{\rm NS} \\
C_{(2)} &= B_{\rm NS} \\
F_0 &= \frac{e^{\Phi} L_{\rm NS}^2}{2\pi\alpha'} (\cos\chi_0 \cosh\psi_0 \cosh\omega \, d\omega \wedge d\tau + \sin\chi_0 \sinh\psi_0 \sin\theta \, d\theta \wedge d\phi) \,. \qquad (3.1)
\end{aligned}$$

The string metric is also changed by this transformation: since the Einstein metric $g_E = e^{-\Phi/2} g$ is invariant, the string metric becomes $ds^2 = e^{\Phi} ds^2_{\rm NS}$. In what follows, we redefine $L^2 \equiv L^2_{RR} = e^{\Phi} L^2_{\rm NS}$.

3.2 Quadratic fluctuations

Since the S-duality transformation is a symmetry of the equations of motion deduced from the low-energy action, it is expected that the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ content of the spectrum be the same as in the previous section. Anyway, it is useful to check explicitely that S-duality works properly in that case.

To do this, we expand the action (2.3) in the fluctuations as in section 2.3. As in the NS case, the linear terms disappear.

At the first look, the quadratic terms look messy, since $\delta\psi$, $\delta\chi$, $f_{\theta\phi}$ and $f_{\omega\tau}$ are coupled in all possible ways, contrary to the NS case. However, by taking

$$\begin{aligned}
\delta_A &= \sinh\psi_0 \cos\chi_0 \, \delta\psi - \cosh\psi_0 \sin\chi_0 \, \delta\chi \\
\delta_S &= \cosh\psi_0 \sin\chi_0 \, \delta\psi + \sinh\psi_0 \cos\chi_0 \, \delta\chi \qquad (3.2)
\end{aligned}$$

the quadratic terms read

$$\begin{aligned}
\delta^{(2)} S &= T_{D3} L^4 e^{-\Phi} \frac{\cosh\psi_0 \sin\chi_0}{\sin^2\chi_0 + \sinh^2\psi_0} \int d\omega \, d\tau \, d\theta \, d\phi \sqrt{-\det\gamma} \times \\
&\times \Big[-\frac{1}{2} (\partial_m \delta_A \partial^m \delta_A + \partial_m \delta_S \partial^m \delta_S) - \frac{1}{4} f_{mn} f^{mn} + (\delta_A)^2 - (\delta_S)^2 - \\
&\quad - 2\delta_A \frac{f_{\omega\tau}}{\cosh\omega} + 2\delta_S \frac{f_{\theta\phi}}{\sin\theta} \Big] + T_{D3} L^4 e^{-\Phi} \frac{\sinh\psi_0 \cos\chi_0}{\sin^2\chi_0 + \sinh^2\psi_0} \int f \wedge f \,. \qquad (3.3)
\end{aligned}$$

The similarity with equation (2.6) implies that the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ content of the spectrum is the same, as expected.

As is well-known, in the NS case, the exact spectrum, as determined from CFT, has a brane-dependent cut-off: the maximal allowed spin is half of the magnetic charge p. If S-duality is true, we expect that in the RR case the maximal allowed spin is half of the electric charge. Such a result could not be found directly, since we have no sigma-model for the world-sheet of strings with a nonzero RR background.

3.3 The effective metric

As is well-known, for any D-brane without RR backgrounds, the fields on the brane can be considered as coupled to an effective metric (open string metric) given by the formula

$$g_o^{-1} = \left[(g + \mathcal{F})^{-1} \right]_S \qquad (3.4)$$

where the index S means symmetrizing the matrix. In the case of the $AdS_2 \times S^2$ D-brane in $AdS_3 \times S^3$, it was noticed in [2] that both radii are equal to L independently of the position of the brane, although the induced radii depend on it and can be very different of each other.

It is interesting to see whether there is, in the RR case, a notion of effective metric with such properties. From the fluctuations found in the previous section, it is clear that the effective geometry is $AdS_2 \times S^2$ with equal radii. The absolute normalization of these radii cannot be deduced from the previous calculations only.

A way to find it is to T-dualize our setup along a dimension orthogonal to $AdS_3 \times S^3$ to obtain an $AdS_2 \times S^2 \times S^1$ D-brane in $AdS_3 \times S^3 \times S^1$. The nonzero resulting background fields are the following (with x the additional dimension):

$$\begin{aligned} ds^2 &= ds^2 \left(\mathrm{AdS}_2 \times \mathrm{S}^2\right) + dx^2 \\ C_{mnx} &= C_{mn} \left(\mathrm{AdS}_2 \times \mathrm{S}^2\right) \end{aligned} \tag{3.5}$$

the dilaton is, as previously, a constant, and the electromagnetic field is the same. It is then easy to see that the WZ term of the action is left unchanged (except for the fact that T_{D3} is replaced by T_{D4}), so that all additional terms come from the determinant in the DBI term.

After some straightforward calculation, one finds that the quadratic terms of the action are essentially the same as in eq. (3.3), except for the following facts:

- T_{D3} is replaced by T_{D4}.
- $f \wedge f$ is replaced by $f \wedge f \wedge dx$.
- More importantly, new terms are added inside the bracket:

$$L^2 \left(\sin^2 \chi_0 + \sinh^2 \psi_0\right) \left[-\frac{1}{2}(\partial_x \delta_A \partial^x \delta_A + \partial_x \delta_S \partial^x \delta_S) - \frac{1}{2} f_{mx} f^{mx}\right]. \tag{3.6}$$

From these new terms, one immediately obtains that the absolute normalization of the $AdS_2 \times S^2$ effective radius is

$$R^2 = L^2(\sin^2 \chi_0 + \sinh^2 \psi_0). \tag{3.7}$$

It is easy to check that this is what one finds from the formula (3.4) without any correction.

So, the situation is different from the NS case, since the radius depends on the position of the brane, and can take any value, which seems to contradict S-duality. However, as S-duality is a symmetry of the equations of motion, and not of the action, this is not surprising.

In particular, in the limit $\psi_0 \to \infty$, one obtains an infinite radius, i.e. flat space. The question whether this could lead to a realistic brane universe would certainly be interesting to investigate.

Acknowledgments

I thank C. Bachas for long discussions and carefully reading the manuscript. I also thank S. Ribault and P. Windey for useful discussions.

A. SL$(2,\mathbb{R})$ symmetry of the D3-brane

We include here, for the reader's convenience, the proof of the SL$(2,\mathbb{R})$ symmetry of the low-energy effective action of the D3-brane. We essentially follow the lines of [5, 6, 7].

For the antisymmetric symbol, we use the convention $\varepsilon_{0123} = \varepsilon^{0123} = +1$. (With this convention, ε is *not* a tensor.)

The Born-Infeld and Wess-Zumino terms of the action read:

$$S = T_{D3}\left(-\int d^4x\sqrt{-\det(\hat{g}_E + e^{-\Phi/2}\mathcal{F})} + \int \hat{C}_{(2)}\wedge\mathcal{F} + \frac{1}{2}\int C_{(0)}\mathcal{F}\wedge\mathcal{F}\right)\,, \tag{A.1}$$

where $\mathcal{F} = 2\pi\alpha' F + \hat{B}$ and $g_E = e^{-\Phi/2}g$ is the Einstein metric. The equation of motion can be written $d\tilde{F} = 0$, where

$$2\pi\alpha'\tilde{F}_{mn} = \varepsilon_{mnpq}\, e^{-\Phi/2}\,\frac{\partial L_{BI}}{\partial\bar{\mathcal{F}}_{pq}} + \hat{C}_{mn} + C_{(0)}\mathcal{F}_{mn}\,, \tag{A.2}$$

$$\frac{\partial L_{BI}}{\partial\bar{\mathcal{F}}_{pq}} = \frac{1}{2}\sqrt{-\det(\hat{g}_E+\bar{\mathcal{F}})}\,[(\hat{g}_E+\bar{\mathcal{F}})^{-1}]^{[pq]} \qquad \text{with } \bar{\mathcal{F}} = e^{-\Phi/2}\mathcal{F}\,. \tag{A.3}$$

We want to show that there is an SL$(2,\mathbb{R})$ symmetry acting as:

$$\begin{pmatrix} C_{(2)} \\ B\end{pmatrix} \to \begin{pmatrix} r & -s \\ -t & u\end{pmatrix}\begin{pmatrix} C_{(2)} \\ B\end{pmatrix}; \quad \begin{pmatrix} \tilde{F} \\ F\end{pmatrix} \to \begin{pmatrix} r & s \\ t & u\end{pmatrix}\begin{pmatrix} \tilde{F} \\ F\end{pmatrix};$$
$$\tau \to \frac{r\tau+s}{t\tau+u}\,, \qquad \text{where } \tau = C_{(0)} + ie^{-\Phi}\,. \tag{A.4}$$

To do this, we consider the infinitesimal transformations

$$\begin{aligned} \delta C_{(0)} &= 2aC_{(0)} + b - c\left(C_{(0)}^2 - e^{-2\Phi}\right) \\ \delta\left(e^{-\Phi/2}\right) &= e^{-\Phi/2}\left(a - cC_{(0)}\right) \\ \delta\mathcal{F} &= -a\mathcal{F} + c\tilde{\mathcal{F}} \qquad \text{with } \tilde{\mathcal{F}} = 2\pi\alpha'\tilde{F} - \hat{C}_{(2)} \end{aligned} \tag{A.5}$$

and we want to show that

$$\delta\tilde{\mathcal{F}} = a\tilde{\mathcal{F}} + b\mathcal{F}\,. \tag{A.6}$$

The b term is trivial.

Noticing that

$$\delta\bar{\mathcal{F}} = ce^{-\Phi/2}\left(\tilde{\mathcal{F}} - C_{(0)}\mathcal{F}\right) \tag{A.7}$$

it is easy to show that the a term is what we expect.

After a straightforward calculation, the c term reads

$$\delta_c\tilde{\mathcal{F}}_{mn} = c\left(e^{-3\Phi/2}\varepsilon_{abcd}\,\frac{\partial L_{BI}}{\partial\bar{\mathcal{F}}_{cd}}\,\varepsilon_{mnpq}\,\frac{\partial^2 L_{BI}}{\partial\bar{\mathcal{F}}_{ab}\partial\bar{\mathcal{F}}_{pq}} + e^{-2\Phi}\mathcal{F}_{mn}\right). \tag{A.8}$$

Using the commutativity of the partial derivatives and the symmetry properties of the ε symbol, we obtain

$$\delta_c \tilde{\mathcal{F}}_{mn} = c \left(\frac{1}{2} e^{-3\Phi/2} \varepsilon_{mnpq} \frac{\partial}{\partial \bar{\mathcal{F}}_{pq}} \left(\varepsilon_{abcd} \frac{\partial L_{BI}}{\partial \bar{\mathcal{F}}_{ab}} \frac{\partial L_{BI}}{\partial \bar{\mathcal{F}}_{cd}} \right) + e^{-2\Phi} \mathcal{F}_{mn} \right) . \tag{A.9}$$

The antisymmetric part of $(\hat{g}_E + \bar{\mathcal{F}})^{-1}$ reads

$$\left[\left(\hat{g}_E + \bar{\mathcal{F}} \right)^{-1} \right]^{[pq]} = \left[\left(\hat{g}_E + \bar{\mathcal{F}} \right)^{-1} \right]^{pp'} \bar{\mathcal{F}}_{p'q'} \left[\left(\hat{g}_E + \bar{\mathcal{F}} \right)^{-1} \right]^{qq'} \tag{A.10}$$

and one has, for any matrix M,

$$\varepsilon_{abcd} M^{aa'} M^{bb'} M^{cc'} M^{dd'} = \varepsilon^{a'b'c'd'} \det M \tag{A.11}$$

so that

$$\varepsilon_{abcd} \frac{\partial L_{BI}}{\partial \bar{\mathcal{F}}_{ab}} \frac{\partial L_{BI}}{\partial \bar{\mathcal{F}}_{cd}} = -\frac{1}{4} \varepsilon^{abcd} \bar{\mathcal{F}}_{ab} \bar{\mathcal{F}}_{cd} \tag{A.12}$$

and finally $\delta_c \tilde{\mathcal{F}} = 0$, which is what we want.

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